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Variational methods have been developed to treat problems in ordinary and partial differential equations involving multibump solutions, especially of homoclinic and heteroclinic type. Methods have also been obtained to find solutions of Hamiltonian systems heteroclinic to a pair of periodic solutions.

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SOME PROBLEMS IN NONLINEAR ANALYSIS

Final Report

Paul H. Rabinowitz

April 6, 1994

U.S. Army Research Office
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The goal of this research program was the development and application of methods from the calculus of variations to problems in differential equations. In terms of methods, there were two major contributions. The first was a variational method for constructing so-called multibump solutions to problems on unbounded domains for ordinary and partial differential equations. In the simplest settings, a "single bump" solution is one which is small (e.g. exponentially) outside of a bounded set and a "multibump" solution is roughly a sum of single bump solutions, the distance between bumps being large compared to the diameter of the bounded regions. The method we use involves variationally putting together one bump solutions to get multibump solutions. In that sense it is reminiscent of shadowing arguments in the theory of dynamical systems.

Applications of this method have been made, jointly with V. Coti Zelati, to superquadratic Hamiltonian systems establishing the existence of multibump homoclinic solutions [2]. Moreover, together with Coti Zelati and work recently completed, multibump periodic solutions have been obtained. Jointly with F. Giannoni [7], we have obtained multibump solutions for a class of Hamiltonian systems on a manifold. In another direction, there have been applications to Hamiltonian systems of multiple pendulum type obtaining homoclinic and heteroclinic solutions near heteroclinic chains [4]. Finally there have been applications, jointly with Coti Zelati, to semilinear elliptic partial differential equations on $\mathbb{R}^n$ getting multibump solutions of homoclinic type [5].

The second main method we have found is a method for finding solutions of Hamiltonian systems which are asymptotic as $t \to \pm\infty$ to a pair of periodic solutions [8], [10]. This involves a novel variational formulation of the problem. In work in progress a variant of the method suitable for treating semilinear elliptic partial differential equations on cylindrical domains has also been obtained and applied in particular to some problems that involve water waves in an infinite strip.

The remaining papers treated standing wave solutions of a nonlinear Schrödinger equation [3] and a method for finding periodic solutions of prescribed energy for singular Hamiltonian systems [11]. There is also an extensive survey paper [9] on variational methods and their applications to differential equations.

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[6] On a model problem of compound pendulum type, to appear, Proc. Int. Conf. on Variational Methods.

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[8] Heteroclinics for a reversible Hamiltonian system, 18 pages, to appear Ergodic Theory and Dynamical Systems.

[9] Critical point theory and applications: a survey, 53 pages, to appear in book with tentative title "Nonlinear Analysis".

[10] Heteroclinics for a reversible Hamiltonian system, 17 pages, to appear Differential and Integral Equations.

[11] A note on period solutions of prescribed energy for singular Hamiltonian systems, accepted, J.C.A.M.